Sym City!

Primary Objectives

Students will:

- Recognize lines of symmetry
- Classify polygons
- Identify reflections (flip), rotations (turn), and translations (slide) transformations (symmetry) by name.
- Create symmetrical repeating patterns.
- Design and analyze simple tilings and tessellations
- Name, and describe a variety of shapes (i.e. circles, squares, triangles, rectangles, hexagons, trapezoids) shown in various positions.

Modification Tips for Different Grade Levels:

Kindergarten is primarily concerned with the teaching of bilateral/reflection symmetry. This type of symmetry is also known as mirror imagery, basic symmetry. They need to know that an object that has line symmetry can be folded in half and the two sides match each other, whether vertical, horizontal and diagonal lines of symmetry are used. Kindergartners also need to learn and practice positions of objects: inside and outside, left, middle, and right, top, middle, and bottom, above and below.

1st Grade—Students should be learning the basics of shape symmetry as well as flip, turn, and slide symmetry as well as continuing to work on spatial sense: left, middle, and right, above, below, top, middle, and bottom.
2nd Grade—Student continue to build on their knowledge of repeating patterns and basics of shape symmetry as well as flip, turn, and slide symmetry.
3rd Grade—Students need to build their knowledge of the basics of shape symmetry (are the two sides of an object the same) and know the terms reflection, rotation, and translation symmetry are other words for flip, turn, and slide transformations.
4th Grade—Continue to expand on student's knowledge of geometric patterns as well as rotational symmetry, lines of symmetry, and identifying different planar shapes.
5th Grade—Students continue to practice and learn reflection, rotation, and translation, and lines of symmetry and rotational symmetry, identifying planar figures, numbers of sides in polygons.
6th Grade—Students need to know reflection, rotation, and translation, lines of symmetry and rotational symmetry, and how to classify quadrilaterals.
7th Grade—Students need to identify lines of symmetry, classify polygons, and identify reflections, rotations, and translations. Option: have students graph the images and find their symmetrical coordinates.
8th Grade and above—Students need to identify lines of symmetry, classify polygons, and identify reflections, rotations, and translations, option: have students graph the images and find their symmetrical coordinates.

Examples of Possible Standards to Incorporate:

Kindergarten:

- 6.3.2 Recognize attributes (such as color, shape, size) and patterns (such as repeated pairs, bilateral symmetry).
- 6.3.1 Identify, duplicate, and extend simple number patterns and sequential and growing patterns.
- 6.3.1 Use a variety of manipulatives (such as connecting cubes, number cards, shapes) to create patterns.
- 6.3.2 Name, copy, and extend patterns.
- 6.3.3 Translate simple patterns into rules.
- 6.4.1 Interpret and describe the physical world with geometric ideas and vocabulary.
- 6.4.2 Use positional terms to specify locations with simple relationships.

- 6.4.1 Identify, name, and describe a variety of shapes (i.e. circles, squares, triangles, rectangles, hexagons, trapezoids) shown in various positions.
- 6.4.6 Identify positions (such as beside, inside, outside, above, below, between, on, over, under, near, far, forward, backward, top, middle, bottom, left, right) using models, illustrations, and stories.

1st Grade:

- 6.4.1 Recognize, describe, and draw geometric figures.
- 6.4.2 Compose and decompose geometric shapes.
- 6.4.1 Recognize and describe similarities and differences between 2-dimensional figures (geometric attributes and properties).
- 6.4.2 Recognize 2- and 3-dimensional figures from different perspectives and orientations.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.3.1 Find repeating patterns . . .

2nd Grade:

- 6.1.13 Use manipulatives such as pattern blocks, tangrams, etc. to explore geometric concepts of symmetry and transformations.
- 6.4.1 Recognize, classify, and transform 2- and 3-dimensional geometric figures.
- 6.4.2 Reflect, rotate, and translate shapes to explore the effects of transformations.
- 6.4.1 Describe common geometric attributes of familiar plane and solid objects.
- 6.4.7 Investigate and describe composition, decomposition, and transformations of polygons.

3rd Grade:

- 6.4.2 Understand and apply the concepts of congruence and symmetry.
- 6.4.4 Identify, create, and describe figures with line symmetry.
- 6.4.2 Classify polygons according to the number of their sides and angles.
- 6.4.1 Describe properties of plane figures (such as circles, triangles, squares and rectangles) and solid shapes (such as spheres, cubes and cylinders).
- 6.4.1 Recognize polygons and be able to identify examples based on geometric definitions.
- 6.4.3 Identify the line of symmetry in a two-dimensional design or shape.

4th Grade:

- 6.4.17 Predict the results of a transformation of a geometric shape.
- 6.4.18 Determine whether a geometric shape has line and/or rotational symmetry.
- 6.4.19 Design and analyze simple tilings and tessellations.
- 6.4.20 Draw lines of symmetry in 2-dimensional figures.
- 6.4.21 Recognize two-dimensional faces of three-dimensional shapes.
- 6.4.10 Identify images resulting from reflections, translations, or rotations.

5th Grade:

- 6.4.7 Understand, select and use units of appropriate size and type to measure angles, lengths/distances, area, surface area and volume.
- 6.1.1 Given a series of geometric statements, draw a conclusion about the figure described.
- 6.1.1 Make and test conjectures about geometric properties and develop logical arguments to justify conclusions.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.

6th Grade:

- 6.4.1 Understand and use basic properties of triangles, quadrilaterals, and other polygons.
- 6.4.2 Use the concepts of translation, rotation, reflection, and symmetry to understand congruence in the plane.
- 6.4.7 Work with transformations in a plane and explore their meanings through drawings and manipulatives.
- 6.4.10 Describe the effect of a transformation on a 2-dimensional figure and the resulting symmetry.

7th Grade:

- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.

8th Grade:

- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.

High School: Geometry

- 8.4.7 Apply the major concepts of transformation geometry to analyzing geometric objects and symmetry.
- 8.4.8 Establish processes for determining congruence and similarity of figures, especially as related to scale factor, contextual applications, and transformations (symmetry).
- 4.32 Recognize, identify and apply types of symmetries (point, line, rotational) of two- and three- dimensional figures.
- 4.33 Use transformations to create and analyze tessellations and investigate the use of tessellations in architecture, mosaics, and artwork.

- 4.34 Create and analyze geometric designs using rigid motions (compositions of reflections, translations, and rotations).
- 8.4.10 Identify, describe, and/or apply transformations on two and three dimensional geometric shapes.

Examples of Possible Academic Vocabulary to Incorporate:

Kindergarten:

- Addition
- Afternoon
- Classify
- Compare
- Date
- Difference
- Hour
- Location
- Minus
- Morning
- Number
- Order
- Pattern
- Position
- Shapes
- Sort
- Subtraction
- Sum
- Time
- Today
- Tomorrow
- Yesterday
- Zero

1st Grade:

- Data
- Digit
- Direction
- Equal to
- Estimate
- Even
- Graph
- Greater than/less than
- Half-hour
- Horizontal
- Length
- Measure/measurement
- Minute
- Odd
- One-half
- Part
- Plus
- Ruler
- Symbol
- Total
- Vertical
- Week
- Whole

2nd Grade:

- Dimensions
- Distance
- Equivalent
- Extend
- Inch
- Interpret
- Likely/unlikely
- Meter/centimeter
- One-fourth
- One-third
- Outcome
- Pound
- Quarter-hour
- Reflect
- Rotate
- Second (time)
- Set
- Symmetry
- Table
- Transformations
- Transitive
- Translate
- Unknown

3rd Grade

- Angle
- Area
- Array
- Capacity
- Conclusion
- Congruent
- Conjecture
- Intersecting lines
- Inverse relationships
- Line of symmetry
- Line, line segment
- Multiples
- Parallel
- Perpendicular
- Pictograph
- Polygon
- Reasonableness

4th Grade

- Accuracy
- Acute
- Chance
- Composite
- Computation
- Convert
- Coordinate system
- Diameter
- Equation
- Expression
- Face of a polyhedron
- Measures of central tendency (mean, median,
- mode)
- Obtuse
- Pattern rules
- Quadrant
- Radius (pl. radii)
- Range
- Relationship
- Right
- Tiling/tessellation
- Vertex (pl. vertices)

5th Grade

- Convex polygon
- Data collection methods
- Edge
- Inequality
- Irregular
- Justify
- Model
- Parallelogram
- Polyhedral solid
- Rational numbers
- Regular (Platonic) solid
- Remainder
- Round
- Solution
- Surface area
- Variable
- View

6th Grade

- Circumference
- Degree (angles)
- Dilation
- Equiangular
- Equilateral
- Interior/exterior angles
- Isosceles
- Odds
- Protractor
- Pyramid
- Random
- Ratio
- Scalene
- Similarity
- Simulation
- Triangle

7th Grade

- Intercepts
- Property
- Proportional relationships
- Construction
- Polygons
- Bisect (bisector)
- Combination

- Variation
- Deductive & inductive reasoning
- Simulations

8th Grade

- Adjacent angles
- Alternate exterior angles
- Alternate interior angles
- Complementary angles
- Corresponding angles
- Infinite
- Series
- Legs of a triangle
- Line of best fit
- Supplementary angles
- Transversal
- Vertical angles
- Vertical line test

High School:

Instructors, please note that though there are no specific Academic Vocabulary lists for high school students, they will be expected to be familiar with and understand the key mathematics terms and concepts covered within the following lesson. Familiarize yourself with the proper terms for all of the following concepts and make sure that you use them with and explain them to your students.

Sym City!

Everything from butterflies, to algebra, to the universe is based on symmetry. So how come we know so little about it?

Symmetry is the ability to take a shape and match it exactly to another shape. Compared to other famous concepts of mathematics and physics—infinity, uncertainty, relativity—the notion of symmetry might seem a bit boring. Things look the same as their reflection in the mirror—big deal!

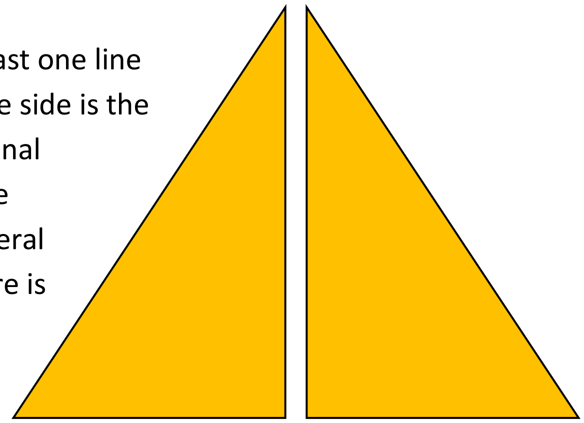
But symmetry conditions our understanding of the universe more completely than any of these other ideas. It would not be far off to say that our basic understanding of what the universe is depends, fundamentally, on the symmetries we find in it.

Symmetry is everywhere you look in nature. If you look at plants and animals, you will find that they have symmetrical body shapes and patterns. If you divide a leaf in half, you will often find that one half has the same shape as the other half.

There are several different types of symmetry, but in each type of symmetry, characteristics such as angles, side lengths, distances, shapes, and sizes are maintained and we are going to use some great activities to introduce students to them:

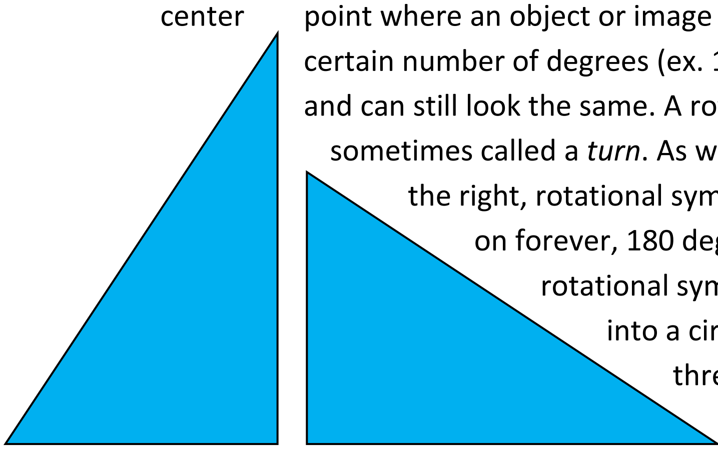
Reflectional Symmetry

An image or object has reflectional symmetry if there is at least one line (vertical or horizontal) that splits the image in half so that one side is the mirror image, or exactly the same as the other side. Reflectional symmetry is also sometimes called line symmetry, mirror symmetry, or bilateral (two sides) symmetry because there is a line in the figure where a mirror could be placed and the figure would look the same. A reflection is sometimes called a *flip*.



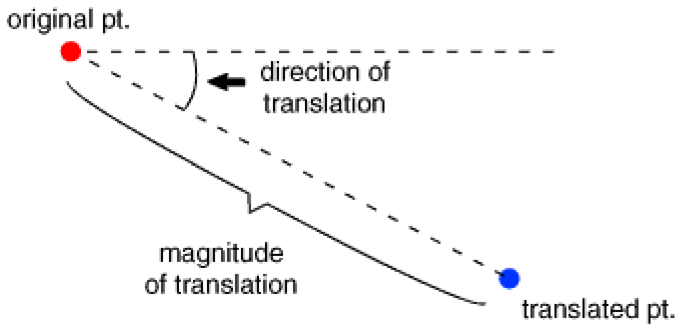
Rotational Symmetry

An image or object has rotational symmetry if there is a center point where an object or image is turned a certain number of degrees (ex. 180 degrees) and can still look the same. A rotation is sometimes called a *turn*. As we can see to the right, rotational symmetry can go on forever, 180 degree rotational symmetry, bent into a circle, with three repetitions of the word infinity.



Translational Symmetry

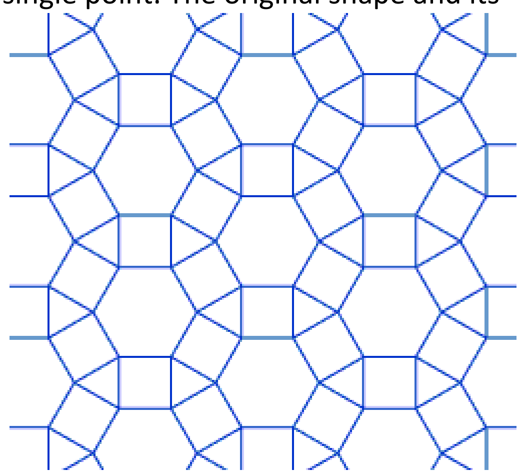
Image Credit:
http://library.thinkquest.org/16661/background/symmetry.1.html



An image has translational symmetry if it can be divided by straight lines into a series of identical figures. Translational symmetry results from moving an object or image a certain distance in a certain direction, also called translating (moving) by a vector (length and direction). What does all of that mean? Translation is just a fancy term for "move." When a shape is moved, two specifications are needed: a **direction** and **magnitude**. Direction can be measured in degrees (e.g., 30 degrees north of east), while magnitude can be measured in inches (e.g., 2 inches) or some other unit of length.

In the image above we can see a simple translation of a point (red) to form another point (blue). Two specifications are needed: direction and magnitude

Translations can be applied to entire shapes, not just a single point. The original shape and its translated copies are said to have translational symmetry.



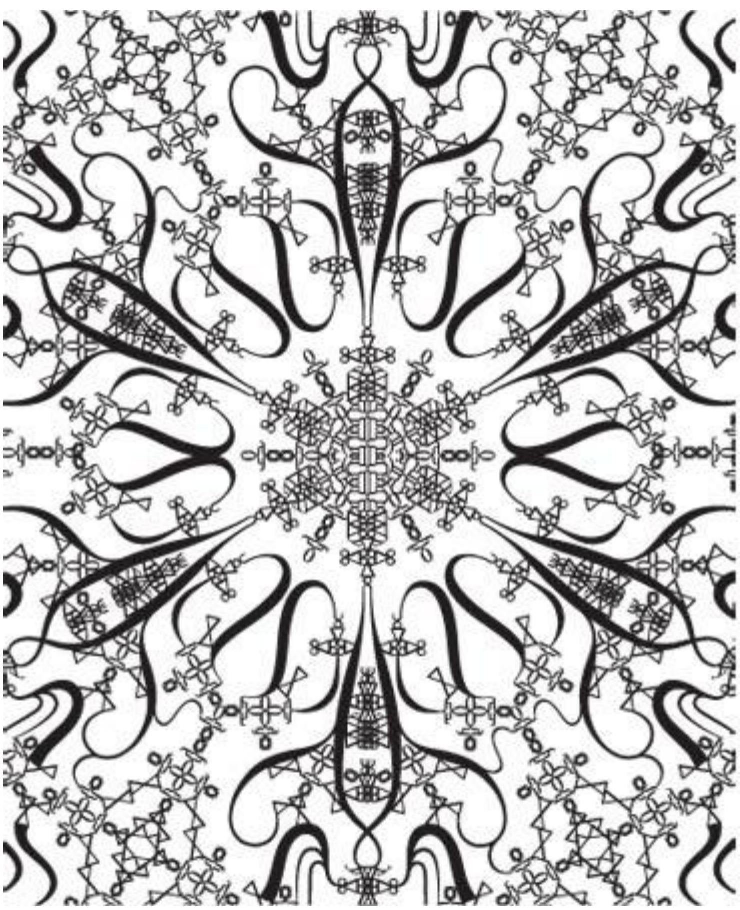
Basically, translation is nothing more than making a copy of a object or point, and then moving it.

Thus, a translation is sometimes called a *slide*. A great example of translation symmetry is in a tessellation; after moving a copy of an image in a certain direction and with a certain magnitude, you find that the copy matches exactly the original.

Now that we know the basic kinds of symmetry…we're going to have a lot of fun putting them to use…remember during each project to discuss the types of symmetry you are using and to apply those standards appropriate to the student's grade levels.

Mirror Drawing

Image credit: http://seedmagazine.com/content/article/a_festival_of_likeness/ Marian Bantijes



This fun, challenging activity is appropriate for young children all the way up to adults and is a great introduction to symmetry. It is trickier than you might think to mirror someone else's motions, especially when they make curves and diagonals. Encourage students to draw slowly, so one person doesn't get too far ahead of the other, and have students trade who leads and who follows from time to time.

The following handout has been designed to fit on an ordinary 8.5" by 11" sheet of paper, but this activity works even better if you use larger sheets of paper. Large sheets of newsprint, or craft paper, work especially well.

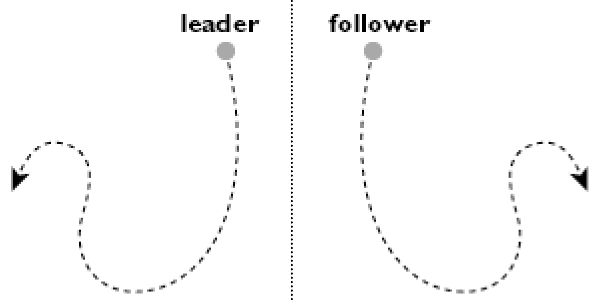
The resulting drawings are often quite beautiful.

Materials:

This is a game for two people. Each needs a pencil or a pen. One will lead, and the other will follow.

Leader: sit on the left.
Put your pencil on your dot.
Slowly start tracing
the dotted arrow with your pencil.
Keep drawing on your side of the paper.
Draw anything you want.
Move slowly so the follower can follow you

Follower: sit on the right.
Put your pencil on your dot.
Follow the leader.
Trace your arrow with your pencil.
Keep following, as if looking in a mirror.
Be sure your pencils are always
the same distance from the middle line.
< < < Try trading who leads and who follows > > >


leader
follower

Planet Symmetria!

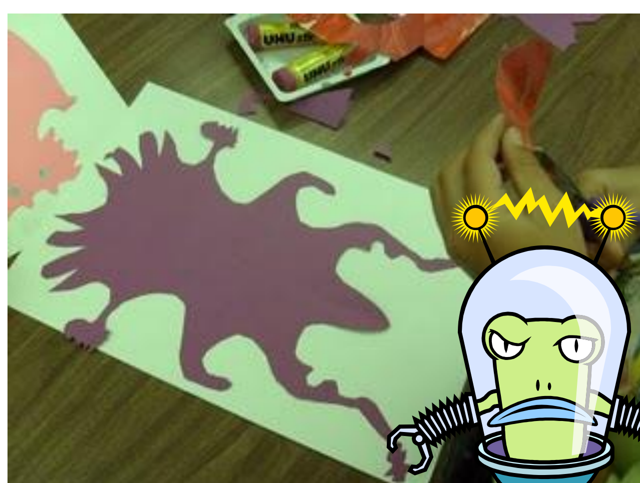
Alien Art project inspired by Patty Palmer at http://deepspacesparkle.blogspot.com/2009/07/symmetrical-paper-cut-aliens.html. Go there for amazing art ideas and projects.

There is nothing more pleasing to the ear than hearing a six year old explain symmetry, let alone saying it. It's a fun word...lots of strange syllables. So now turn the word into creepy, strange, whimsical alien and you have pure joy.

Creating the Alien

Not always easy for tiny hands, but you'll be surprised by how determined the smallest student can be. This lesson works with Kindergarten first and third grade students and while the older kids may get more out of the activity the younger kids can learn plenty, too.

Have each student take a sheet of construction paper and fold it in half lengthwise. It doesn't have to be lengthwise, but it'll make the alien taller. It's okay if you have a kid who folds it the wrong way.

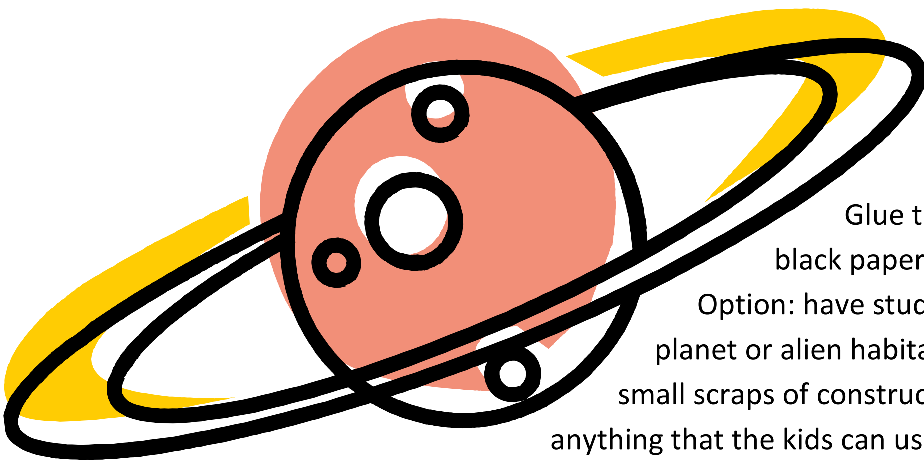


Now, for the next step, you have options...

For younger kids (K-2nd grade), give them a crayon (not a pencil...they'll want to erase again and again and that isn't the point) and have them draw a squiggly line something that resembles a half of a head, half of a body, arms, legs, etc from the top of the paper down to the bottom. Our subject of

symmetry comes in here. Explain how the students will only draw *half* a body. You may need to demonstrate this step repeatedly, but the rest of the project is a breeze.
Once the line is drawn, cut it out. Many kids will cut the wrong side of the paper. They will end up with two pieces instead of one alien. One way to fix it, is to glue the two pieces together. They'll have a line down the middle of their alien, they might think this is a mistake, but you will know that actually this is a perfect moment to discuss symmetry.

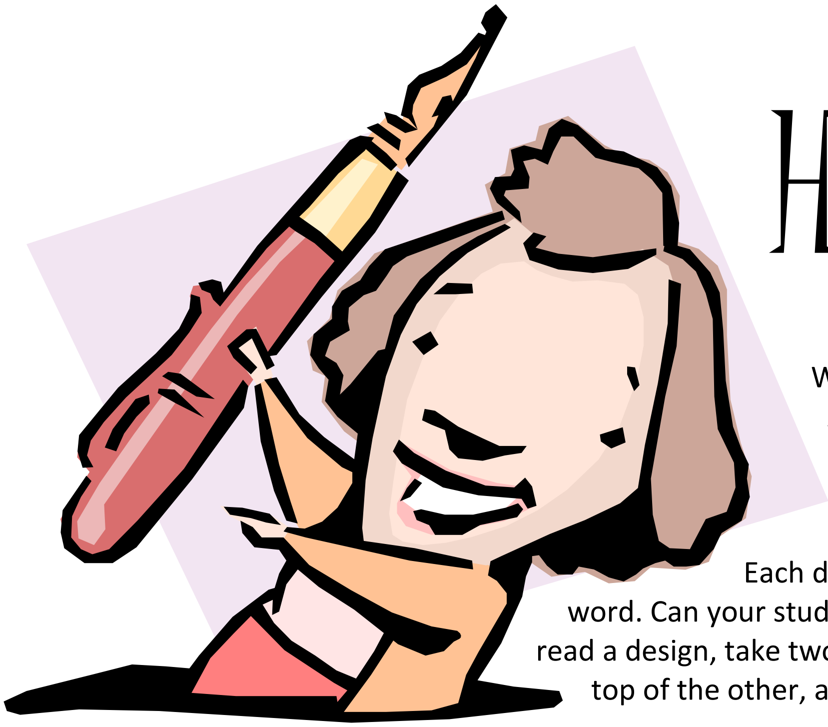
For older kids, give them a pair of scissors and instruct them to create their half of an alien body just by cutting. This ends up to be more of a surprise. When they open up the paper, they will have a big blob. Now it's time to turn that blob into an alien.



Planet Symmetria: Bring Your Alien to Life!

Glue the alien onto a piece of white or black paper, either provides a great contrast. Option: have students color or paint a symmetrical planet or alien habitat on the background paper. Have small scraps of construction paper, sequins, googly eyes, anything that the kids can use to decorate their aliens (remember that what is done to one side must be done to the other).

Option: To top it off, use some glitter and glue for a dazzling effect. Warning: Kids go haywire over glitter glue.



Half Words

An *inversion* is a word or name written so it reads in more than one way.

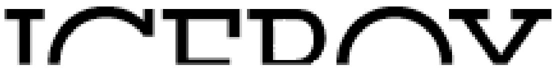
Each design on the following page is really half of a word. Can your students figure out what each design says? To read a design, take two copies of the next page, place one copy on top of the other, and slide them around until the two copies of the design meet. Hold the papers up to a light so you can see through both sheets. You may have to rotate or flip over one of the pages. For instance, the second design makes the word "mirror".

Hand out one copy to each student. Students must then work in pairs to figure out the eight words. Few groups will be able to figure out all the words. Nonetheless students usually like a challenge, and enjoy working together.

Encourage students to explore other shapes they can make by superimposing the designs beyond the ones Scott Kim originally had in mind. Design 3, in particular, makes many different shapes.

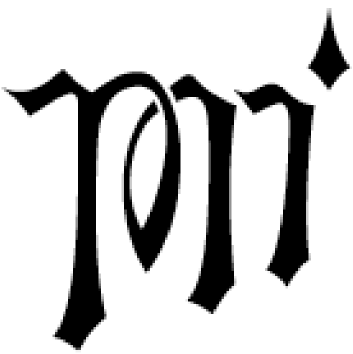
Each design, when assembled properly, makes a perfect inversion. There are three fundamentally different types of symmetries and this is a great way to introduce students to them: 180 degree rotational symmetry, reflective symmetry about a horizontal axis, and reflective symmetry about a vertical axis. One word, #7, has to be turned sideways to be read. Here are the answers…

1.



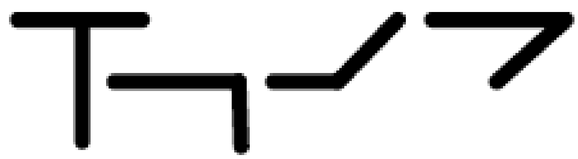
5.

2.



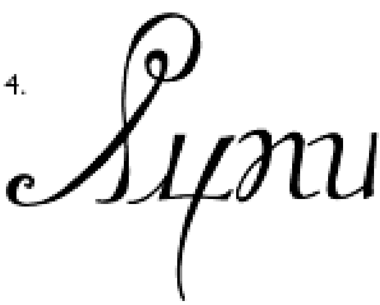
6.

3.



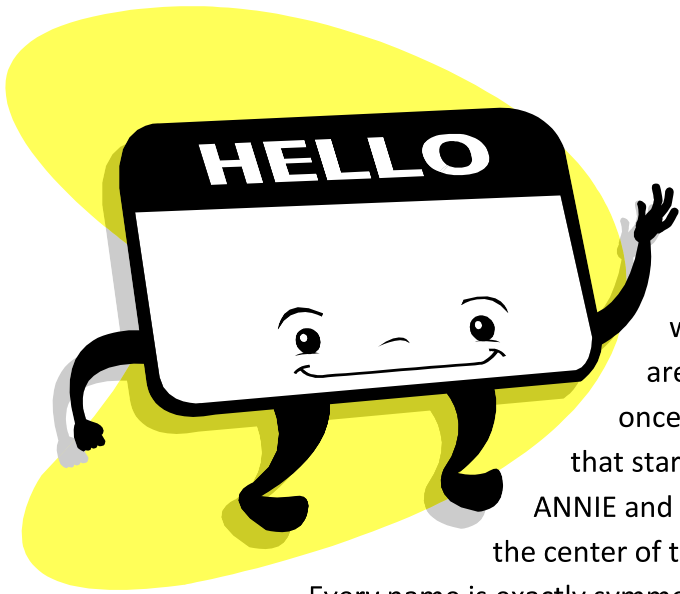
7.

4.



8.

Name Inversions



There are 26 first names on the name sheet, one for each letter of the alphabet. Some names are written in capitals, some in small letters, and some are mixed. Each lettering style occurs exactly twice, once as a boy's name and once as a girl's name. Names that start with letters at opposite ends of the alphabet — ANNIE and ZANE, for instance — appear on opposite sides of the center of the design.

Every name is exactly symmetrical. Most have rotational symmetry, meaning that they look the same right side up Some have reflective symmetry, meaning that they look the same in a mirror. Sometimes the line of symmetry is horizontal; sometimes it is vertical. Only the name OTTO has both rotational and reflective symmetry.

Some students will want to try making inversions out of their own names and the following Name Symmetry project will help give them ideas. Of course if they want to do it on their own they should go for it, keep in mind, some names are easier to invert than others. If a first name doesn't seem to work, suggest trying a nickname, a last name, or a friend's name.

On the following page is a complete list of the names on the sheet and their symmetries. "H Mirror" stands for "mirror reflection about a horizontal line" and "V Mirror" stands for "mirror reflection about a vertical line." The hyphenation shows how letters are grouped. For instance, in the name ANNIE, the first letter A turns into the fifth letter E, but the second letter N does not turn into the fourth letter I: instead, the middle NNI makes one indivisble chunk.

	Name	*Symmetry*	*Same style as*
A	a-nni-e	Rotation	z-a-n-e
B	B-O-B	H Mirror	Y-V-E-T-T-E
C	C-a-n-d-y	Rotation	Q-u-i-n-cy
D	D-E-B-B-I-E	H Mirror	T-I-M-O-T-H-Y
E	E-V-E	V Mirror	w-a-r-r-e-n
R	F-ra-nc-in-e	Rotation	mi-ch-ael
G	G-r-e-g	Rotation	Ire-ne
H	HAN-NAH	Rotation	NAT-HAN
I	Ire-ne	Rotation	G-r-e-g
J	J-u-l-i-a-n	Rotation	L-i-l-y
K	ki-m	Rotation	R-ob-in
L	L-i-l-y	Rotation	J-u-l-i-a-n
M	mi-ch-ael	Rotation	F-ra-nc-in-e
N	NAT-HAN	Rotation	HAN-NAH
O	O-T-T-O	Both	U-N-A
P	P-h-i-li-p	Rotation	v-i-rg-in-i-a
Q	Q-u-i-n-cy	Rotation	C-a-n-d-y
R	R-ob-in	Rotation	ki-m
S	S-u-zan-n-e	Rotation	x-a-v-i-e-r
T	T-I-M-O-T-H-Y	V Mirror	D-E-B-B-I-E
U	U-N-A	Rotation	O-T-T-O
V	v-i-rg-in-i-a	Rotation	P-h-i-li-p
W	w-a-r-r-e-n	Rotation	E-V-E
X	x-a-v-i-e-r	Rotation	S-u-zan-n-e
Y	Y-V-E-T-T-E	V Mirror	B-O-B
Z	z-a-n-e	V Mirror	a-nni-e

INVERSIONS NAMES

Find a name for each letter of the alphabet. Which names are the same upside down? Which names are the same in a mirror? For each name, find another name written in the same style.

Name Symmetry

This project is great for 4th grade students and above. Idea and pictures from http://lc-art-gallery.lcms.srvusd.net/KramerMain.html

Making designs using your names is your student's goal as well a s learning how to color their designs using markers in a different way. Have each started with a square of 8 1/2" thin copy paper. Fold it in half once, twice, and then fold it into small triangles. In one of these triangles draw your name using 5 or 6 letters, filling up the triangle space as much as possible with your letters. Outline the letters in reg. sized sharpies. Open up the triangle and flipped it so the drawn outline shows through to another triangle and then outline the first lettered triangle into the second triangle and so on.

Color in the shapes with markers, and if you want went over the edges of spaces with another color to give them a sort of undulating feel of going in and out with color. Each name will make a different, unique design.

Variation:

Give each student a piece paper and ask them to fold it in half lengthwise. With the paper folded, they are to write their name in cursive, adjusting to fill the paper as much as possible. The bottoms of the letters should always touch the fold of the paper. Any descenders (such as the bottoms of g's, j's or y's) need to be ignored for this project.

Have the students go over the pencil lines with a large black Sharpie. If they also trace the backside, fold, and trace again on the remaining side, they will eventually have one side with reflecting names.

The fun part is to have students turn their drawings into some kind of creature. They can draw details with a thin marker and then color sections in with crayons. Tell the students that whatever color or pattern is made on one side, must also be done on the other.

For younger students:

Take a piece of thick light colored construction paper and fold it in half width-wide, creasing the middle. Then go around with several colors of washable paint and put two globs in the middle of the open paper. Have the kids close the paper and rub it from the crease toward the outside to spread the paint. Then they open it to see what they've created. They can close the paper again and continue to move the paint until they're happy. After they dry have them add touches with markers to create symmetry animals or symmetry faces.

Symmetrical Me! Symmetrical You!

http://www.education.com/activity/article/Draw_the_Other_Half/

Most faces are symmetrical—that is, both sides of the face have similar proportions. In fact, having a symmetrical face is a mark of beauty. The more symmetrical a face is the more beautiful it is considered to be. That's why, throughout the centuries, many works of art featuring faces or designs in paintings, sculptures, and patterns seek to have symmetry. In this activity, inspire your students to explore symmetry with this project that has them practicing art and math measurements by drawing the other half of a given face.

Materials:

- A close-up magazine image or photo of an animal face or a human face. Both sides of the face should be clearly visible.
- White drawing paper
- A ruler
- Scissors
- Glue or tape
- A pencil

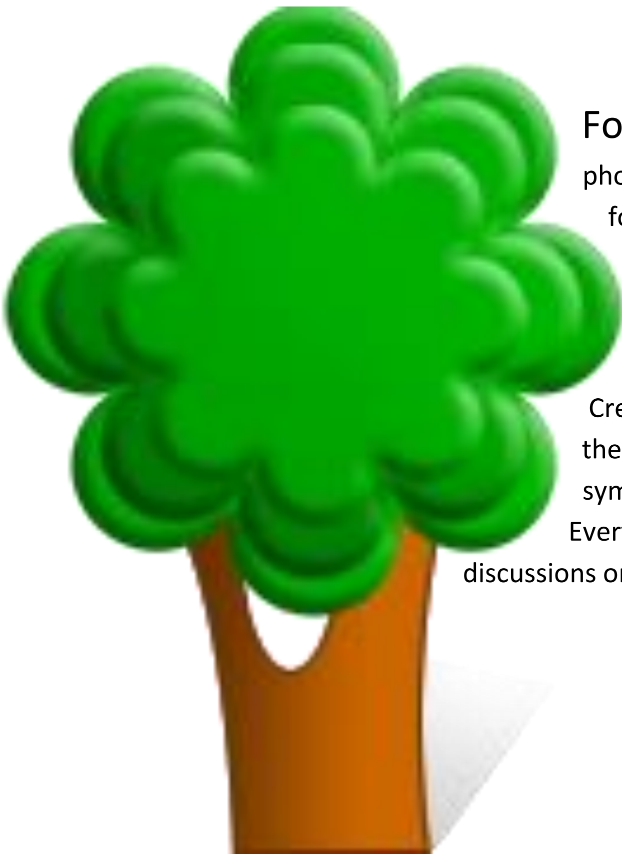
1. Use a ruler to draw a vertical line down the center of the face photo. This is a line of symmetry. Cut along the line to create two halves.
2. Glue or tape one side of the image to a plain sheet of paper. Do not put any tape or glue on the side where you will be drawing. Be sure to leave enough space on the sheet of paper for you to complete the missing half.
3. Before drawing, take a moment to observe the face. To figure out where the facial features you'll be drawing should go, use your ruler to measure the horizontal (sideways) and vertical (up-and-down) distances from the photo's specific features to the line of symmetry. For instance, the distance from the bridge of the nose to the inner corner of the eye would be the same on both sides of the face, so

measuring from the inner corner to the line of symmetry on one side will tell you how far away the line is to the other inner corner.

4. With these measurements, you can map out beforehand where the drawn facial features should go by lightly placing pencil marks where you've calculated them to be.
5. Now start drawing the other half of the face!

When you're done, you should have a beautiful symmetrical face. Try this again with different kinds of faces to check out how features and proportions differ among people, or try it with other symmetrical objects like simple shapes, patterns, and even butterflies!

Option: Take a close up picture of your student's face (either the day before or using a digital...if you have access). Then they cut the picture in half and draw the half of their face that is missing, making a personal connection to math!

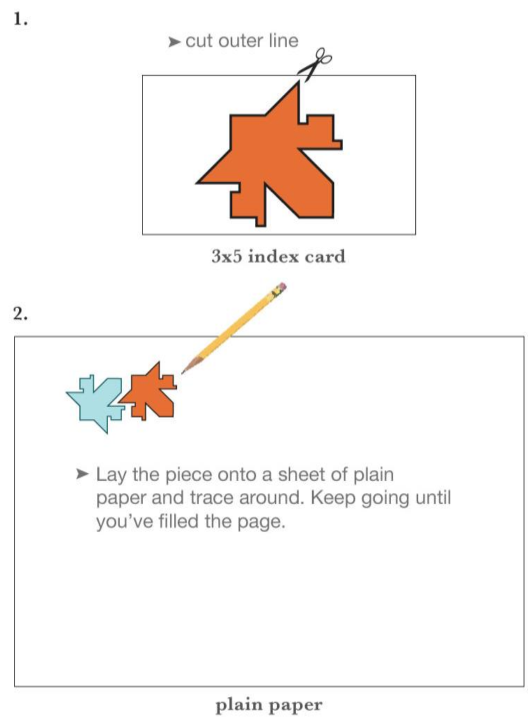


For younger students: Use pictures of people from photographs or magazines, mount them to construction paper for sturdiness and cut to the form of the picture. Then cut them in half. Mix up the cards and place face down. The students have to find the symmetrical match and place the symmetrical objects in a "symme-tree" stack.

Create "Symmetry City." Get a large piece of butcher paper for the background of your city. The students can create anything symmetrical for thei city- people, trees, buildings, etc. Everything has to be symmetrical to belong in your city. Lead discussions on some "non- examples" of symmetry.

Terrific Tessellations

In math, as they advance, students will spend more time exploring more complex shapes than ever. The squares and triangles of the early grades expand into multi-sided figures, many of them three dimensional. In upper grades, these explorations form the basis of advanced geometry. At the beginning, the most important priority is for kids to develop an awareness of how figures work in space.



We have all seen these symmetrical forms in nature—a beehive, for example, is a prime example. Or perhaps you're a fan of M.C. Escher, whose work so famously links to math. M. C. Escher was a Dutch graphic designer who created mathematically inspired symmetrical shapes that joined together like a puzzle. This technique of fitting together multiple shapes without any gaps or overlapping is called *tessellation*. The edges of a tessellation look like the edges of a jigsaw puzzle, and fit together like one as well.

Your student doesn't have to be a naturalist or renowned artist to enjoy tessellations - all it takes is some index cards, some scissors, and some open-eyed imagination!

In this activity, your student will create her own shape to use as a repeating pattern to form a tessellation. Equilateral triangles, hexagons, rectangles and squares will all usually tessellate. But you can even make more complex shapes that tessellate. But making a shape that tessellates isn't as easy as it may seem! This activity is something she can do on her own to get those wheels turning and those creative juices flowing, as your student builds up her analytical and mathematical thinking skills.

Option: http://library.thinkquest.org/16661/background/symmetry.1.html

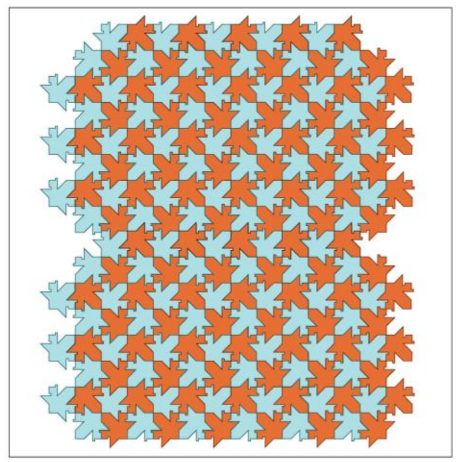
What You Need:

- Plain index cards, 3x5”
- Scissors
- Scotch tape
- Blank white paper

Method 1:

1. Start with one index card, placed horizontally on a table surface.
2. Cut as many edges as you like, as if making a puzzle piece.
3. Don’t put the puzzles back together the same way, though; instead, trace your shape onto a piece of paper, and then "fit" it against that piece to start a patterned design.
4. Keep going until you’ve filled the page, and then don’t hesitate to have fun coloring in the different “puzzle” pieces in different creative ways. Congratulations! you have just started your first “tessellation,” and the future possibilities are endless.
5. In fact, “tessellations” are a fabulous activity for a rainy day. You can challenge the whole family to come up with innovative designs; and if you get one you especially like, you might even want to turn it into a stamp using simple materials like a cut potato, and create cards or wrapping paper. But even if you just explore this “math art” on regular paper, remember this: every exploration builds skills for geometry work in the future. And who knows? Maybe you’ve got a budding architect, artist, or even M.C. Escher on your hands! There’s only one way to find out: grab some cards and scissors, and get started.

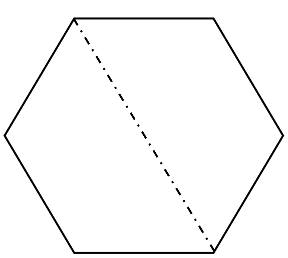
3.

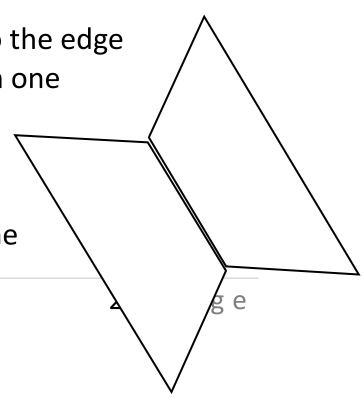


finished

Method 2:

1. Choose a shape (square, rectangle or hexagon). Draw the shape of your choice making sure opposite sides are equal of equal length.
2. When your basic shape is done you can make alterations to the edge of the shape if need be. Draw a line inside the shape from one corner on one side to the other corner on the same side. A new shape will be created as a section of the first shape.
3. Cut out the new shape that was made when drawing a line



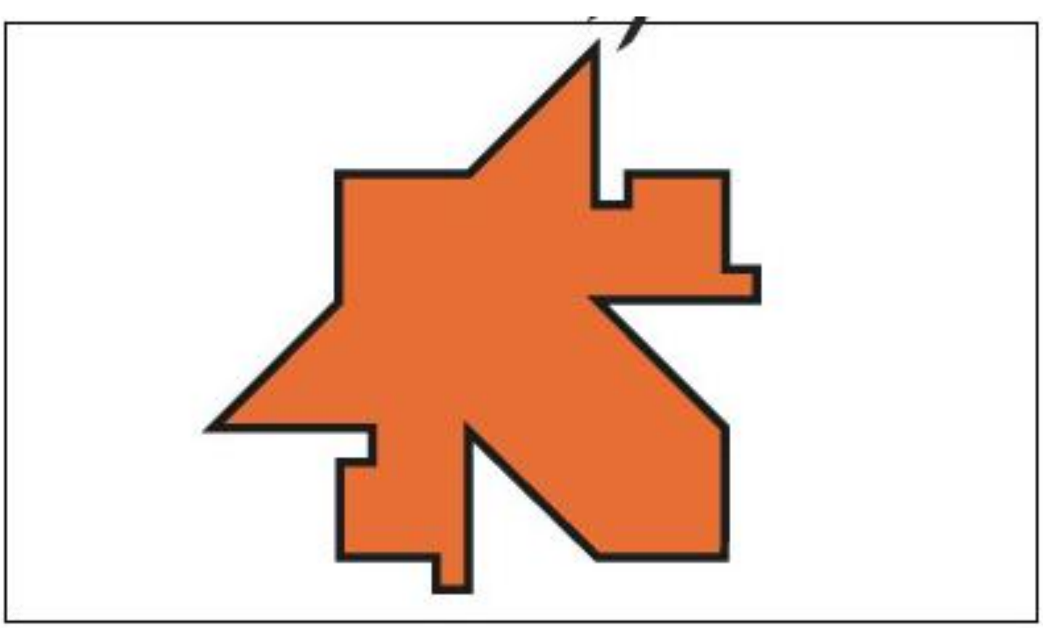


from one corner to the other.

4. Tape this new shape (that is a section of the old shape as a whole) on the opposite side of the old shape that was left when you cut the new section from it.
5. Use this shape as your “tile” and trace around it to create a template. Cut the template out.
6. Test your shape to see if it still tessellates (fits together in a repeating pattern). Cut out more than one copy of your sample tile, being very careful to duplicate the shape as exactly as possible. Lay your duplicates down turning them as needed. Can you interlock the shapes from various sides? If so, you have a successful tessellation tile!
7. Make light parallel lines across the back of your sheet paper. They should be the width of your basic shapes (from external corner to external corner, in other words not the diagonal)
8. Trace your shape several times so that it fits together in a repeating pattern, filling up the entire page with your interlocking shape without any spaces in between.
9. You can use a utility knife to remove the in between shapes so that the remaining shapes form a pattern.
10. Glue a contrasting sheet of paper to the back of your cut out sheet to really make it pop!
11. Enjoy the fruits of your labor! You'll end up with an incredible visual arrangement

Variation: At step 5, after you have your basic shape, make one copy. Make sure they interconnect. What do the shapes suggest? Can you see a profile along the edge? Can you imagine an animal inside the shape? If so, draw it in. Make copies and cut them out. Fit them together and color them as desired. The shapes should contrast either in color or in the drawn details so that when pieced together they flow from one image to another, separating out the different animal forms.

You can see a practical example of tessellations in pieced quilts. Each piece must fit together smoothly in order to create a flat sewn surface with no gaps or holes. What a challenge!



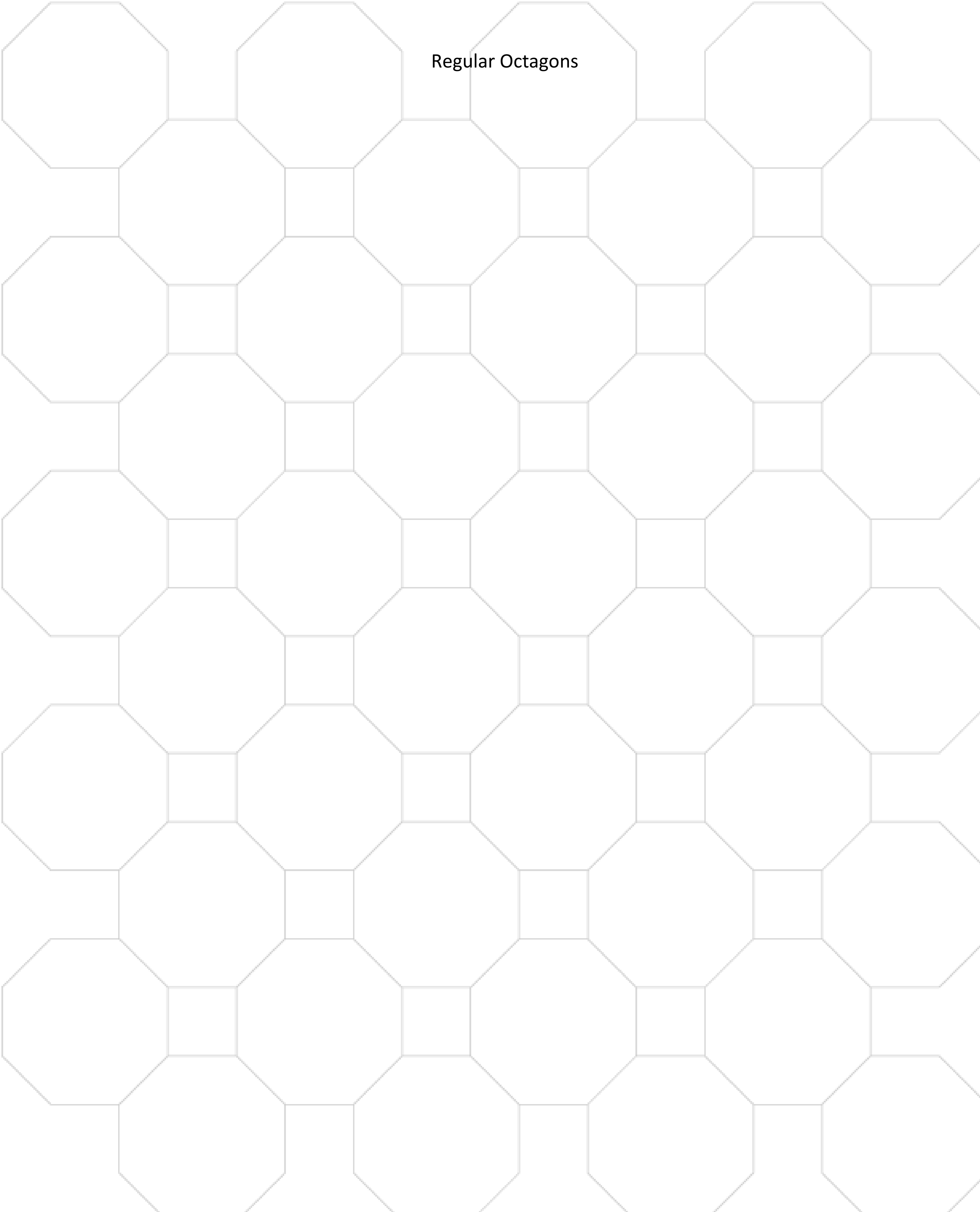
Regular Polygon Patterns for Tessellations

For Creating Equilateral Triangles for Tesselations

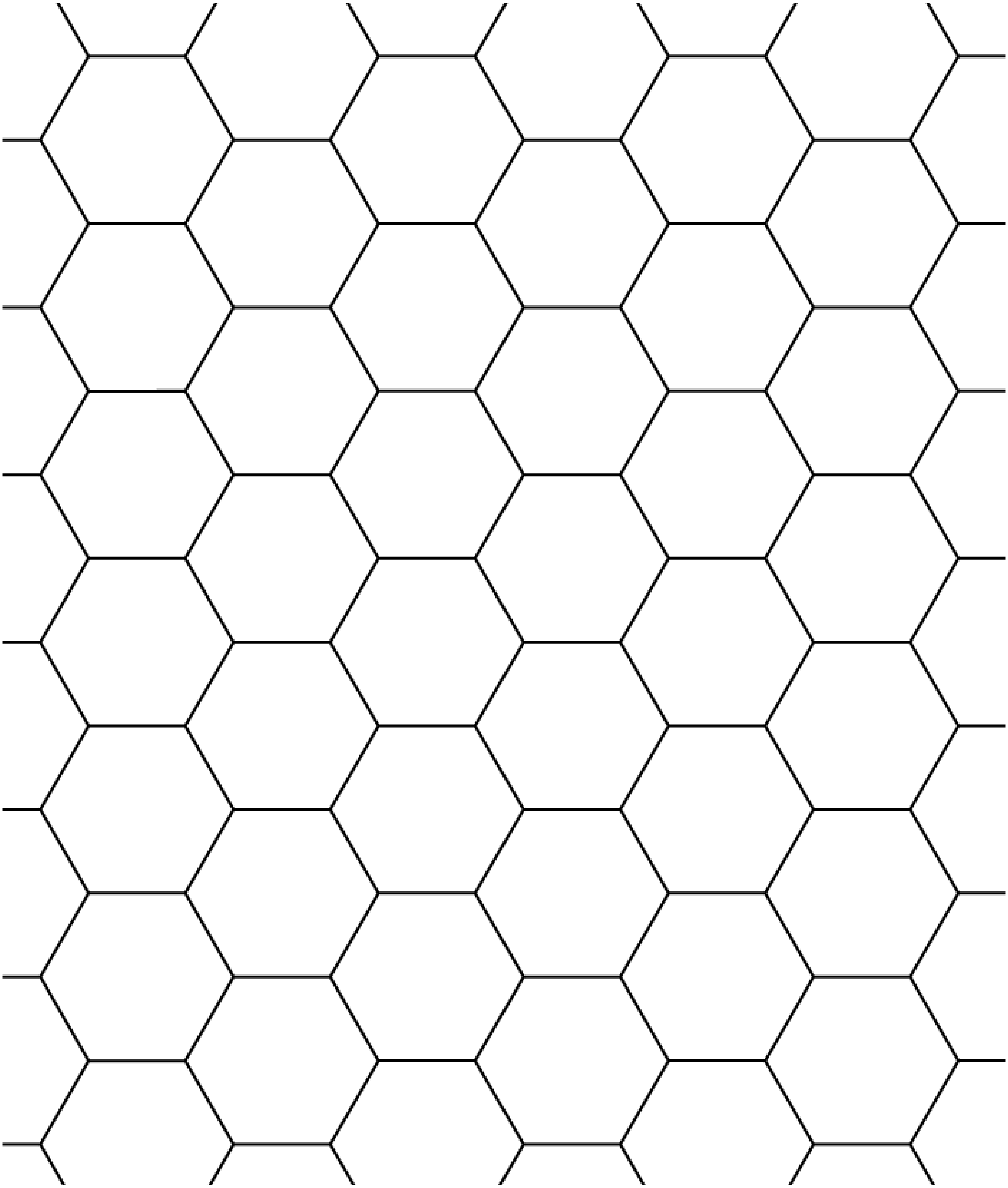
For Creating Equilateral Squares

Regular Hexagons

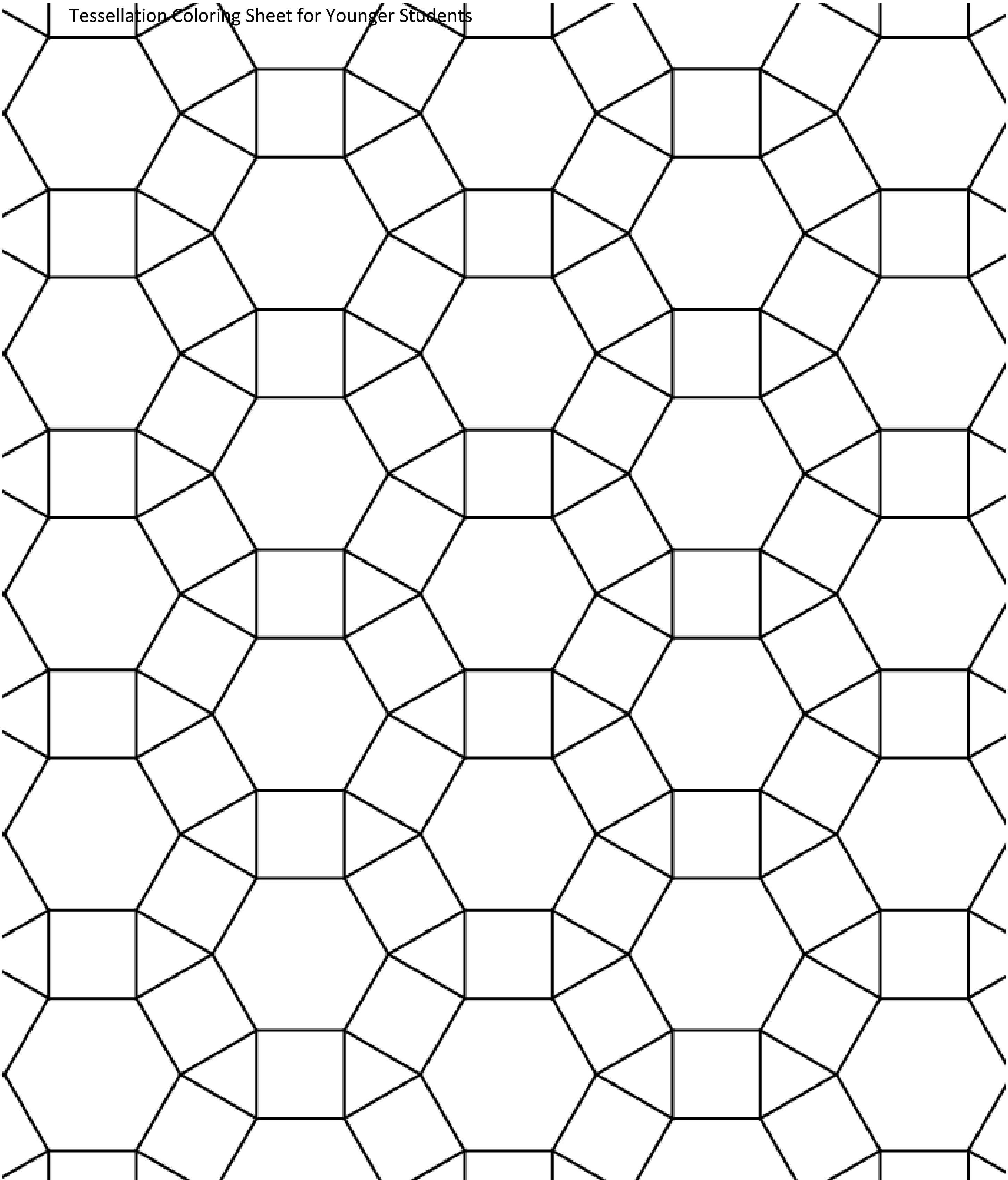
Regular Octagons



Regular Dodecagons



Tesselation Coloring Sheet for Younger Students



Madagascar

Madagascar Solitaire

You can find symmetry everywhere! Even in ancient (but still really fun) games.

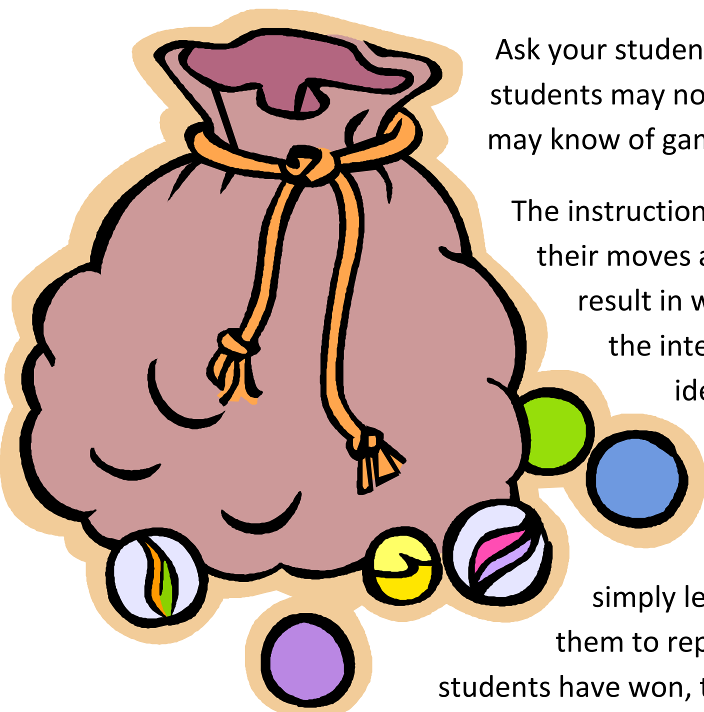
Madagascar Solitaire is an ancient game from the island country of Madagascar. This game encourages players to think about strategy and symmetry. It also teaches skills related to graphing.

Materials:

A handful of stones, beans, paperclips or other small objects to use as markers.

Copies of the boards and score cards for the students

If you have a group of students this game can be played individually or as a group and it's a great game to use in a group where skill levels vary. By using the different game boards you can make this game easier or more challenging and because it is solitaire your students can easily start with a simpler board and move on to more advanced ones when they are ready.



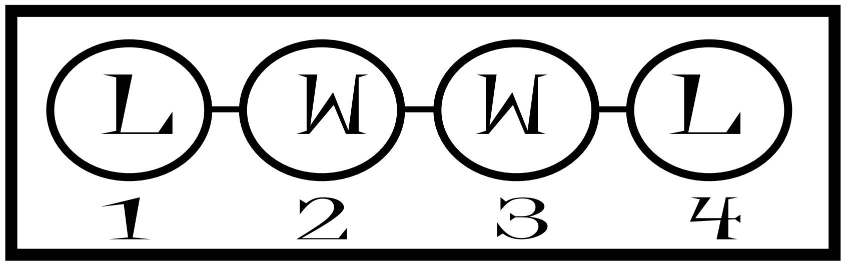
Ask your students if they know any kind of solitaire games. Some students may not be familiar with the term solitaire, though they may know of games like this.

The instructions for the game suggest that players keep track of their moves as they play. This lets them learn what strategies result in winning games and show others how they won. In the intermediate game this also allows students to practice identifying locations using (x,y) coordinates, a graphing skill.

Note: If students find it too frustrating to keep track of their games moves while they are playing simply let students play for a while and when they win, ask them to repeat the moves to see if they can do it again. After students have won, they have much more incentive to record their moves—so that they can show everyone else how they did it. Even players who aren't recording **all** of their moves will find it useful to record their first moves because certain starting moves make it impossible to win!

Tips for Teachers:

In simple solitaire whether a player wins this game depends on which marker the player takes away first.



For an example look at the image. A player will always lose if they start the game by removing the markers in spots 1 or 4, but will win if they start in the middle, in spots 2 or 3. It's symmetry.

The two winning moves are the same move, if you look at the board mathematically. Since it is symmetrical, removing 2 has the same effect as removing 3 would. They're mirror image moves. It's easy to see the symmetry in Simple Solitaire, so looking for the line of symmetry doesn't help much, but where symmetry really comes into play is when students move on to more advanced boards. Now remember, it's possible to lose a game even if you start with a winning first move.

Strategy:

Strategy involves looking ahead and thinking about what will happen after each move, how that move will change what you can do next. Thinking ahead helps you win more often. Now, the point of the game isn't just to win, but to be able to show others what moves help you win and why. Challenge your students to win a game, and then demonstrate how they did it.

Does symmetry really help?

Winning moves are mirror images of each other. Just like in Simple Solitaire, where the winning moves are mirror images of each other, it is the same in Intermediate Solitaire. You can win if you start by removing the marker at 4,1. Now imagine that the lines of symmetry on the board are like a mirror. What is the mirror image of that move? Can you find multiple mirror images of that move? A player can win by starting with this move and playing a game in which every move is a mirror images of the move in the winning game.

You may want to have some students work in pairs, with one student playing, and the other keeping track of the moves. Players can use a score card to track their moves. Other students may want to play until they win, and then repeat the moves that led to their win and record them.

For students that have a hard time getting started, draw a large group game board and use sticky notes for markers, so you can all play together.

The Goal of the Game:

Remove all the markers from the game board one by one by jumping over them with other markers. You win when you only have one marker left, alone on the board.

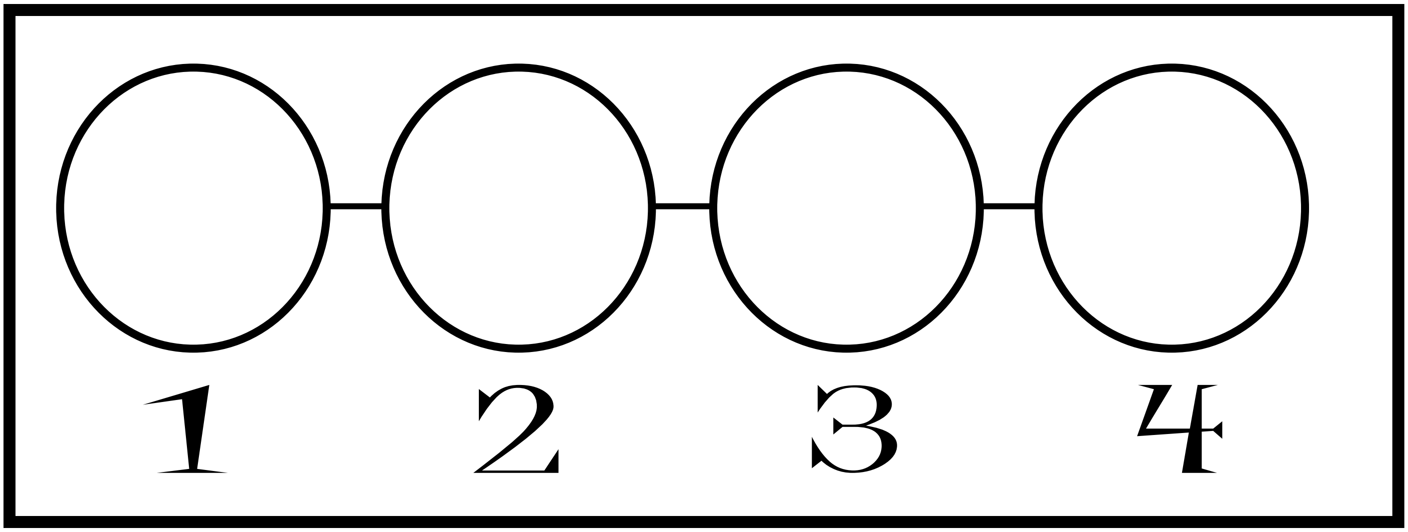
How to Play:

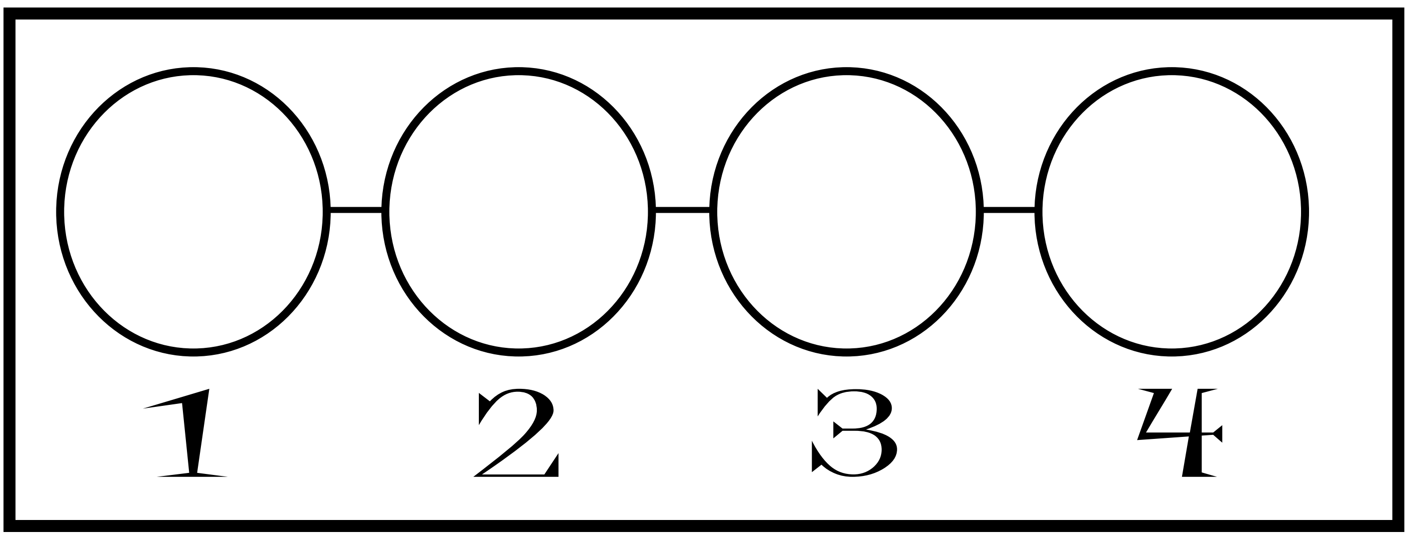
1. Place a marker in every circle on your game board.
2. Remove one marker from your board. In classic Madagascar Solitaire, on the large board, it is the center circle that remains empty.
3. Jump one marker over another. You can't jump over multiple markers, you can only jump over one marker at a time and you have to jump over a marker, you can't jump over an empty space. And you must **land** in an **empty** circle.
4. After you jump over a marker, you take it off of the board. Then, jump again, using any marker, not only the one you just used.
5. If you end up with only one marker left on the board, you win! More than one marker left on the board and no jumps possible, try again!

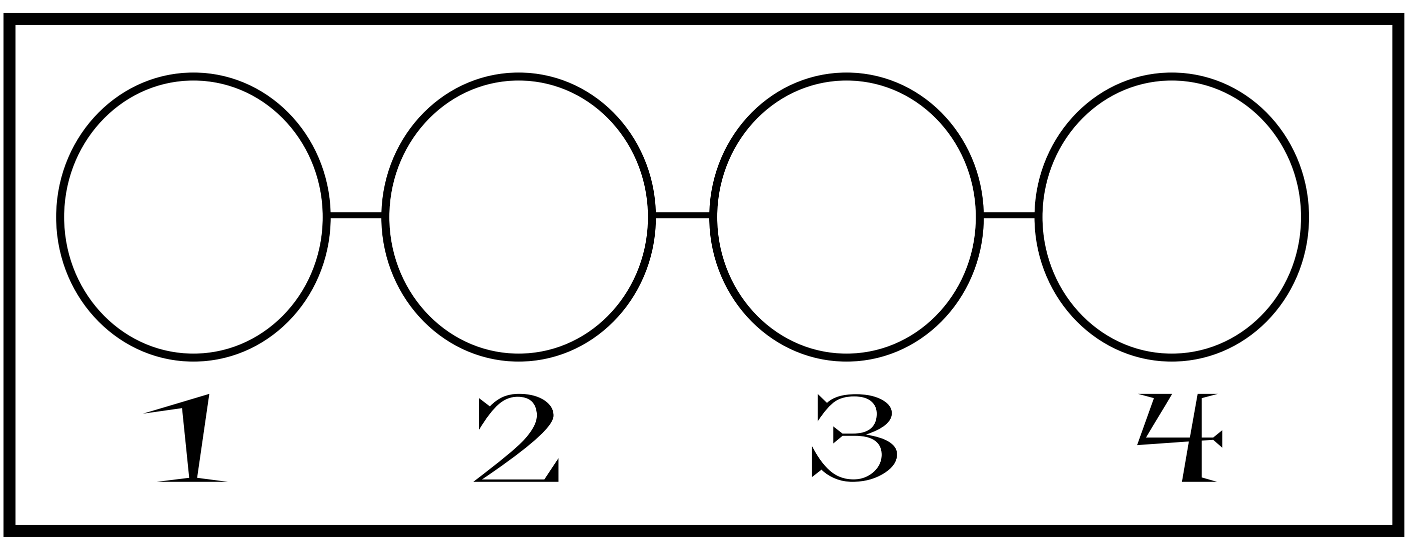
Let's Practice! Simple Solitaire

1. Have students play a game using the following 4 circle board, keeping track of their moves on the basic scorecard. If they won have them put a star on their scorecard. (This isn't just because you won, it's to help them start noting what moves lead to a winning game.)
2. Have them play again, starting with a new marker. Can they figure out what marker they need to start with in order to win?

3. After they figure out one winning move, can they figure out another? How many can they find? Are the winning moves similar? Have them always put a star next to a winning game.


1 2 3 4


1 2 3 4


1 2 3 4

Game #	
Marker Moved	New Position

Game #	
Marker Moved	New Position

Game #	
Marker Moved	New Position

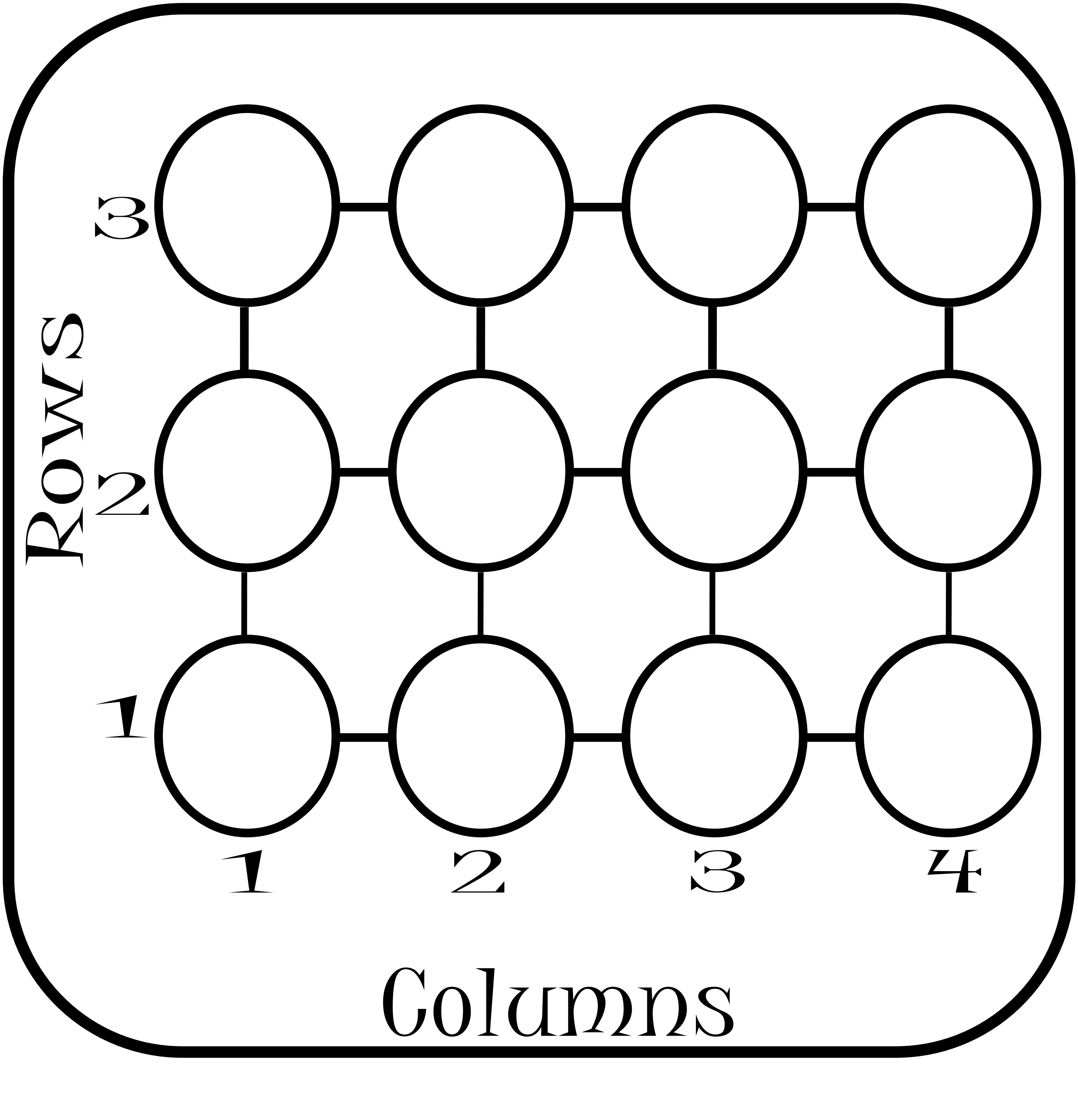
Game #	
Marker Moved	New Position

Getting More Advanced! Intermediate Solitaire

This version has more possible moves than our previous practice did. Markers can jump up and down or left and right, but they still can't jump diagonally.

1. Have students play a few games, keeping track on their scorecards. Once again they will put a star by any games they win.
2. Now our concept of symmetry comes into play once again. This scorecard is symmetrical. When you fold it the circles on one side will match up with the other side. That fold marks a line of symmetry, each side is a mirror reflection of the other side of the board. There are two lines of symmetry on the board.
3. When students win a game have them see if they can win the game by using the symmetrical, mirror reflection move of the first move they made in their winning game.

Note: On the scorecard when students write down a column and row, they are using two numbers to identify an exact position. In math class teachers talk about (x,y) coordinates. On our game board the x-coordinate is the column number and the y-coordinate is the row number.


Rows
3
2
1
1
2
3
4
Columns

Marker Moved		New Position	
Column	Row	Column	Row

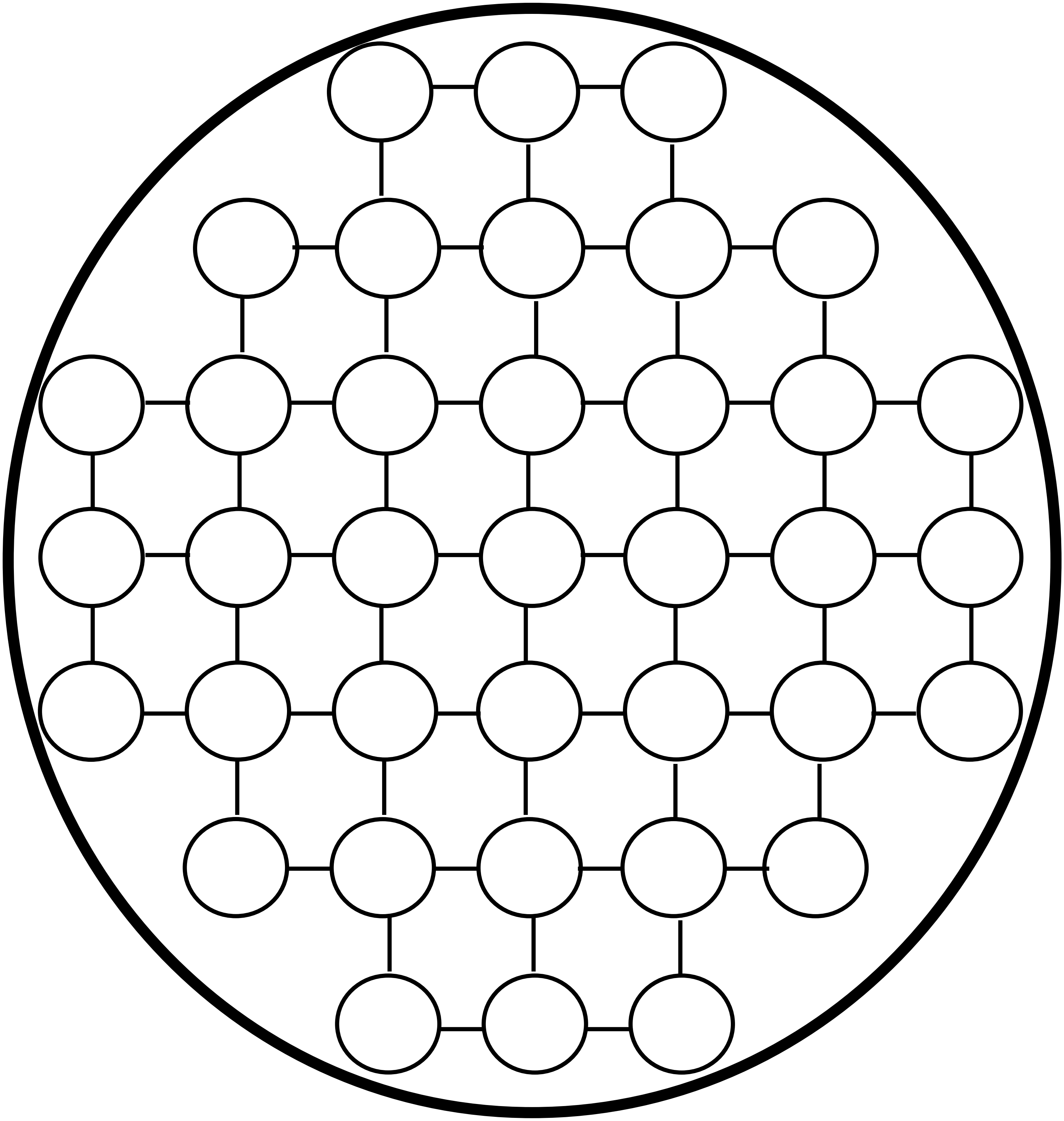
Marker Moved		New Position	
Column	Row	Column	Row

Now You're Ready! Advanced Solitaire

Madagascar Solitaire can be played on many different game boards and now you can give students a chance to design their own.

They may want to try a board with four columns and four rows or a board with six columns and six rows, it just needs to stay symmetrical, what lines of symmetry can they find? Then they can play a game on whatever board they've designed.

For an extra fun challenge have them use the board on the next page, the one used in Madagascar.

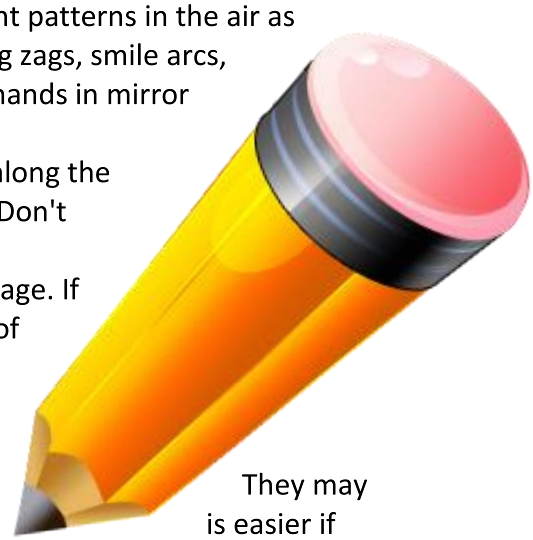


Symmetry Extension Activities

Two-Handed Writing

This activity makes a good warmup for Mirror Writing. Each student needs two pencils or pens, one in each hand, and a large sheet of paper in front of them. Notebook paper will do, but larger paper is better.

Stand up. Following the teacher, move their hands in small circles in the air, being careful not to hit their neighbor. Follow the teacher as he or she makes different patterns in the air as if conducting an orchestra: circles one way, circles the other way, zig zags, smile arcs, rainbow arcs, and figure eights. Feel the sensation of moving their hands in mirror symmetry.



Now each student lean over and let the tips of their pencil drag along the paper as they continue to move their hands in opposite directions. Don't worry about what their drawing looks like; just enjoy the motion.

If they are right handed, place both pencils at the center of the page. If their are left handed, place their pencils at the left and right edges of the page. Now with both hands, at the same time, write their first name in opposite directions. Their normal writing hand will write their name forwards and their other hand will write it backwards.

This may sound impossible to do, but it is easier than it sounds. They may find that cursive is easier than printing. They may also find that it is easier if they don't look at what they are writing, but instead focus on the movement of their hands. Don't be disappointed if they are not able to do this, for some people this exercise is too hard. But many people find this exercise much easier than they expected. They can check their work by holding their paper up to the light and looking through the back of the paper.

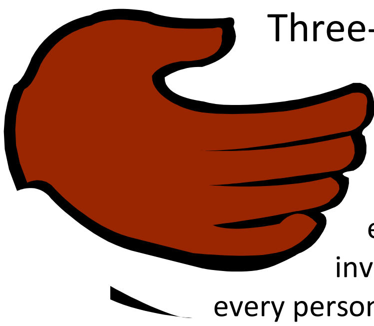
Hiding Behind a Symme-Tree

This activity makes a good followup to Mirror Drawing. The idea is the same, except the two students draw in rotational symmetry about a point instead of reflective symmetry about a line.

This is a game for two people. Each of them needs a pencil or a pen. One of them will lead, and the other will follow.

Tell your students to draw a dot in the middle of the page and pretend this is a tree growing out of the page. The leader will place the tip of their pencil on the paper somewhere near the dot. The follower will place the tip of their pencil on the opposite side of the dot, and at the same distance, as if they were trying to hide from the other person behind the tree.

The leader will slowly start moving their pencil. The follower will follow the leader, making sure their pencil always stays on the opposite side of the dot, and at the same distance. The leader will draw anything they want, being careful to move slowly. They can trade who leads and who follows if youlike.



Three-Person Handshake

Have students stand in groups of 3, facing each other. Groups of 4 are also okay, but groups of 2 are too small. It may help to push desks to the edge of the classroom or move to a bigger space. Ask each group to invent a handshake for all people in the group to do together in which every person does exactly the same thing. For instance, if one person crosses right arm over left, then all three people must cross the same way.

Have each group perform their handshake for the rest of the class. It is easier for people to see if everyone sits down except the performing group.